

Pedagogical Content Knowledge: An Example from Secondary School Mathematics

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Introduction

Despite the passage of 15 years, Shulman's (1987) seminal work on pedagogical content knowledge remains important because of the acknowledgement it gives to an old teacher adage: "You don't know a subject until you've taught it." This bit of folk wisdom suggests that although content knowledge is important for teaching (Anderson, 1989; McDiarmid, Ball, & Anderson, 1989; Ball, 1993) there is a special (and perhaps different) way of knowing that is crucial for effective teaching. Shulman's introduction to the literature of the term *pedagogical content knowledge*¹ provided a framework which could allow researchers and scholars to explore more systematically the intuitive notion that expert teachers possess a kind of deeper and richer understanding that permits them to transform more effectively their own knowledge into forms that are accessible to students.

This line of inquiry speaks to the *kind* of knowledge that teachers have as opposed to the *quantity* of knowledge, and seeks to further refine our notions of teacher knowledge as it relates to pedagogy. Research in this area has focussed on an examination of what teachers know about elementary mathematics that might be useful for the purposes of teaching, using the various dimensions of pck as tools to assess this knowledge (Lowery, 2002; Fuller, 1997). Only a limited number of studies of this kind have been done with a focus on secondary mathematics, specifically in the area of functions and graphs (Evan, 1993; Ebert, 1993), and more generally in the way that prospective teachers are able to construct the kinds of rich connected knowledge that pck requires (Wood, 1993b; Wood, 1993c; Wood, 1994). Teacher educators have also found this way of looking at knowledge transformation through the development of pck to be a useful (but complex) guide to understanding how to develop pedagogical expertise in trainee teachers (Kichan, 2002; Barnett, 1991; Onslow, Beynon, & Geddis, 1992; Geddis, 1993; Geddis & Wood, 1997; Wood & Geddis, 1999).

Research on pck in secondary mathematics is rare. However, as the mathematics education community strives to find a way to move mathematics instruction at all

¹ Consistent with the literature, in this paper I use pck to refer to pedagogical content knowledge of teachers while PCK is used to denote the pedagogical content knowledge of teacher educators

levels closer to the vision conceptualized in reports produced by the National Council of Teachers of Mathematics (NCTM, 1989; NCTM, 1991; NCTM, 2000), a problem remains - the same problem that was apparent to scholars more than a decade ago as they began to look at this newly articulated view of knowledge. Despite attempts by some scholars to develop some sense of what pedagogical content knowledge is really like (Lampert, 1986; Ball, 1993), comparatively little has been done to accurately analyse and describe what this special amalgam of pedagogy and content might look like in various subject domains, especially at the secondary level. As Marks (1990) has pointed out,

[B]ecause pck derives from other types of knowledge, determining where one ends and the other begins is difficult. The attempt to classify instances of teacher's knowledge by type proves to be ambiguous ... (p. 8)

In this paper I shall attempt to illuminate some of the important aspects of pck in the area of secondary mathematics through a rich description of a pedagogical incident that took place while a beginning teacher was teaching a lesson on integral exponents. I first describe a critical incident in the teaching of a mixed ability grade nine Canadian mathematics class that was experienced by a beginning teacher whom I was observing. The unforeseen difficulties experienced by this novice teacher in teaching a particular piece of apparently elementary mathematical content provide the background for a discussion of the pedagogical possibilities nested in this content. This analysis allows for an in-depth discussion of how the interaction of pedagogical knowledge and content knowledge together causes a metamorphosis and fusion of both of these knowledge types into a new understanding which I claim is an instantiation of this elusive entity that we call pck. This illustration of what pck of this topic might look like also leads to a consideration of how this kind of pedagogical content knowledge on the part of the teacher might resemble, or be different from, the kind of knowledge required by a teacher educator trying to foster such understandings in his or her own students.

The Pedagogical Content Knowledge of Teachers

The Teacher

Julie² is a young woman who recently began teaching in a small high school in a medium sized urban centre. She was excited at the thought of beginning her teaching career because it was something she had always wanted to do and because she was fortunate enough to secure a position in the geographical area that she desired. As a pupil herself she had always been highly motivated to succeed in school, both in her academic studies and her athletic pursuits. An avid athlete, she

² All names used in this paper are gender-same pseudonyms.

had enjoyed her experiences working with children in coaching situations. She had also enjoyed tutoring and helping other students while in university; and, she believed that teaching would be a satisfying and rewarding career.

When I approached her about being a participant in a research project that would study beginning teachers, she agreed to take part without reservation and agreed to keep a log of *critical incidents* that she found interesting, unusual or surprising in her daily teaching. The choice of which incidents to include was left up to her and she was informed that they could be about pedagogy, classroom management or anything else that in her opinion was worthy of further discussion. I went to the school to interview her once a week so that we could talk about these experiences in more detail. These interviews were audio-taped and then transcribed. The transcriptions (along with the critical incident sheets) formed the data for this segment of the study.

Julie approached teaching with enthusiasm and a determination to try to teach her pupils³ in an interesting and vital manner. As one would expect, she experienced the typical culture shock that is characteristic of most novice teachers as they realize that not all of their pupils share their views with respect to the importance of the academic tasks of school. She spoke about her experience in the interview:

I think the kids are surprising. They're ... I don't think you realize until you start working with kids what kind of backgrounds they're coming from, and I just have a very varied bunch I guess you could say, and I find that I spend a lot of my time, you know, doing the discipline part of it and classroom management rather than teaching, and that's, you know, something that's true probably in a lot of classrooms, but I mean, that's not something that I expected.

Despite this initial surprise, Julie appeared to be coping extremely well for a first year teacher. After she had completed her first semester of teaching she used the knowledge and experience that she had gained to help her modify her teaching style to take into account what she had learned about teaching a typical mixed ability grade nine mathematics class.

Julie's teacher preparation program emphasized an active approach to the teaching and learning of mathematics and Julie did well in her teacher education studies. Now that she had her own classroom, she appeared confident enough in her

³In this paper the word "pupil" refers to students in a high school classroom while the word "student" refers to a student teacher engaged in a teacher preparation program.

teaching ability to attempt some non-standard approaches with her mixed ability grade nine mathematics class. In her attempt to use a guided discovery approach to the teaching of negative and zero exponents, Julie encountered a number of surprising occurrences, and this lesson became the focus of our interview that week.

The Lesson

In her lesson Julie decided that the best way to get pupils to understand non-positive exponents was to provide them with a guided discovery exercise in which they were asked to evaluate various powers and then to search for a pattern.

My intentions were to allow students to discover the rules for themselves. The way I did that was to develop a pattern, so I started with $2^5, 2^4, 2^3, \dots$ and they were supposed to discover a pattern ... students were to see a pattern and to use that pattern to come up with the rules for the zero and negative exponents.

A typical chart for such an exercise might look something like the one below along with expected pupil responses:

Power	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
Expected Pupil Response	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

To her surprise, however, Julie found that the patterns the students discovered were not exactly what she had in mind.

First of all, students had difficulty discovering the pattern to start with. They wanted to say that numbers were decreasing by 2 instead of being divided by 2 as they went down, and so they came up with 2^0 is equal to zero, 2^{-1} is -2 , and 2^{-2} is -4 , and I'm not sure whether that's because that's the pattern they were seeing or just because people see 2^0 and they want to say it's 0 and 2^{-1} - multiply them together so they get -2 . I'm not sure but that's what they came up with.

The use of a patterning approach in this lesson was based on the belief that pupils will see that if $2^4 = 16$, $2^3 = 8$, $2^2 = 4$ and $2^1 = 2$ then they will observe that any particular term in the sequence is found by dividing the previous term by 2 and

hence $2^0 = \frac{2^1}{2} = \frac{2}{2} = 1$. By similar logic 2^{-1} would be $\frac{2^0}{2} = \frac{1}{2}$ and 2^{-2} would be

$\frac{2^{-1}}{2} = \frac{\frac{1}{2}}{2} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ and so on. The problem that often arises when using this

method is that although this pattern is one possible example of what pupils might discover, there are others that pupils typically “discover” that are not correct. For example, to get from 4 to 2 (2^2 to 2^1) it is correct to say that you could subtract 2. However, this coincidentally correct result leads the pupils to conclude that they should do the same thing to get from 2^1 to 2^0 . When they do this calculation they “discover” that $2^0 = 0$ which is exactly what the teacher does not want them to think in the first place! And, although it may be true that subtracting 2 does not work for earlier terms in the sequence, pupils at this level often find patterns that do not apply to all the cases under consideration and the idea that $2^0 = 0$ has considerable intuitive appeal to the students. Consequently, although the correct pattern seems clear to the teacher, the pupils have constructed equally compelling patterns (to them at least) that do not lead to the desired generalization.

Another difficulty arises because the results that the correct pattern gives are not in any way intuitive for the pupils. What does an exponent of zero really *mean*? When properly introduced, pupils can easily associate a meaning with the symbols used to denote x^2 . It is not unreasonable to accept that the use of exponents is, like multiplication, a convenient short form. Just as multiplication is a short way of writing repeated addition, exponentiation is just a short way of writing repeated multiplication. Consequently, it is not difficult to understand that x^2 means $x \cdot x$ and x^3 means $x \cdot x \cdot x$. However, using this pattern of understanding does not help in trying to establish a meaning (as opposed to a rule) for x^0 . If x^3 does mean $x \cdot x \cdot x$, or as pupils typically think of it (rightly or wrongly), 3 x 's, then x^0 must mean zero x 's. What could x^0 possibly mean other than 0? A slight adjustment in this thinking can, in fact, result in a meaning for zero exponents and this will be considered later.

The Intersection of Pedagogy and Content

Pedagogical Knowledge

It is useful to examine in some detail those aspects of this lesson that are strictly pedagogical and those that fall into the content category because it is precisely the lack of an intersection between the two to produce pck that caused the difficulty in the lesson. The structure of the lesson itself as a guided discovery was a pedagogical decision based on Julie's understanding of this type of lesson style and an understanding of how to set up a clear set of exercises for the pupils that will lead them to make generalizations about exponents and then to construct their own understandings based on these generalizations.

The decision about how to lead pupils to recognize one kind of pattern over another is also a pedagogical one and depends on the teacher's knowledge of pupils and how they typically react to this type of exercise. Decisions about what questions to ask to help pupils to develop the pattern are also important. Asking "What do you notice?" is clearly not as focused as saying "What operation would you have to perform on each term to get the next term in the sequence?"; and, when Julie taught this lesson a second time she had already decided to change the way that she approached this aspect of the lesson to make her questions more specific and less open to misinterpretation.

Content Knowledge

The knowledge of content that is required on the part of the teacher appears to be elementary and is based on a series of simple and familiar rules about integral exponents. However, even this aspect of the teacher's understanding was problematic. For example, at one point in the interview Julie said that "the point of doing the discovery was showing them why we have to *define* 2^0 that way ..." (emphasis added). I questioned her more about this statement and she concluded (after some thought) that in fact the statement $2^0 = 1$ was not a definition but a logical necessity based on other considerations.

This distinction between what could be called *warranted* knowledge and *conventional* knowledge in mathematics is non-trivial because much mathematical knowledge is often portrayed as arbitrary or "by convention" (a definition is an example of conventional knowledge) and therefore unassailable and without warrant⁴. Julie's approach was designed to get pupils to think about *why* $2^0 = 1$ and not to develop a definition, which is essentially arbitrary. Interestingly, she did realize the distinction when questioned further; but, she still appeared to be influenced by many years of experience in mathematics classrooms (at all levels) where many ideas were presented as definitions rather than warranted knowledge.

Pedagogical Content Knowledge

The way that the knowledge of pedagogy and of content fit together to produce pedagogical content knowledge is also complex. It does require an understanding of the fact that when pupils are given a pattern to examine there may be many

⁴There *are* conventions in mathematics - numbers to the right of zero on the number line are considered to be positive, for example. This system of representation could be changed by agreement and there would be no logical difficulty caused by the change. Other bits of wisdom often presented this way, however, are not just convention and could not be changed by agreement without causing logical inconsistencies in the system. For example, it is not just convention that a negative number multiplied by another negative number is positive, despite the fact that some teachers claim it is.

different ways of interpreting the results and that using powers of 2 may not be as useful as using powers of 10, for example, which map directly onto the decimal number system with which pupils are already familiar. A base of 2 in some ways is the worst possible choice because it is the only non-zero number in the number system where the sum and product is the same—that is, $2 + 2 = 2 \times 2$. This fact is at the root of the pupils' desire to subtract rather than divide to establish the value that comes after 2^2 (or 4) as 2 and then to continue to 0 rather than 1 for the case of 2^0 . Julie herself realized this difficulty as I discussed the lesson with her in the interview: “[N]ow that I think of it, it’s not the best example because you can see that other pattern developing, like with subtracting 2 and stuff ...”. In this statement Julie was indicating that she had acquired a piece of important pck that linked together her knowledge of exponents to a new knowledge of how pupils react pedagogically to certain approaches.

In the next part of the interview we discussed the implications of using the base 10 and how this choice might have changed the lesson. Julie had an intuitive sense of why this base might be a better choice but she did call on me as “the expert”⁵ to develop the ideas in more detail. If a base of 10 had been chosen, pupils could first be asked to recall how the place value system that we use for writing numbers works. In the number 1434.71, for example, the first 1 represents 1×1000 because it is in the thousands place while the second 1 represents 0.01 because it is in the hundredths place. Changing these well established ideas to exponential representation gives that 10^3 is the same as the thousands place, 10^2 is the same as the hundreds place, 10^1 is the same as the tens place and therefore the next power of 10 in the pattern is 10^0 or the ones place. In this case the teacher would not be asking the pupils the value of 10^0 or 10^{-1} but rather having them establish that these exponents would be the next logical ones in a pattern which starts with $10^3, 10^2, 10^1, \dots$ and so on. This pattern is clear and there is really no other possible choice for most pupils. Then they could match the pattern with their previous knowledge about what the first digit before the decimal represents (ones) and what the first digit after the decimal represents (tenths) and conclude that $10^0 = 1$ and $10^{-1} = \frac{1}{10}$.

To the novice, however, the distinction between using 2 as the base of the powers and using 10 as the base is invisible. When I originally asked Julie why she had chosen to use 2 rather than some other number for the base she had no explanation

⁵I was open with Julie at the start and suggested that thinking about one’s practice can be a valuable experience and that there very well could be positive spin-offs for her as a result of this research. Consequently, when she called on me to help her with her thinking I felt obligated to do so and to take on a slightly different role than that of the researcher.

other than the fact that she had used it before. It did not appear from her answer that she thought this consideration to be an important one. Both represent the same underlying mathematical principle and hence are equivalent from a mathematical viewpoint. They are not equivalent from a pedagogical viewpoint, however, because the base 10 allows the teacher to hook into all the other experiences with the place value system that pupils can draw upon in their attempts to understand what is happening with zero and negative exponents. It is this intersection of content knowledge and pedagogical understanding of pupils that governs the choice about what *specific* content to use to generate the concepts in this lesson; and, it was only after teaching the topic that the differences became obvious to Julie.

Another way of looking at the pupils' difficulties is to understand from a pedagogical perspective what the pupils were trying to do when they generated answers to questions such as 2^0 . Pupils typically argue that $x^0 = 0$ because the exponent tells you how many x 's there are and in this case because there are none the answer must be 0!

A closer analysis of this logic shows that the pupils in this case were searching not for an answer but for a *meaning* that could be associated with the symbolism of a zero exponent. The pupils already had a meaning that could be associated with positive exponents and they wanted a comparable meaning for the case of a zero exponent. If teachers had pedagogical content knowledge of this topic they might be able to transform their own understanding of the topic to make a connection with the pupils' prior understanding of meaning in terms of exponents. A rich understanding of exponents would allow teachers to generate a meaning for x^0 that *is* consistent with the pupils' prior understanding of other exponents. This kind of connection is not aimed at producing an answer but at connecting together various pieces of the pupil's understanding.

Teachers wishing to make this kind of strong conceptual link could remind pupils that x^2 means $1.x^2$ or $1.x.x$ and that x means $1.x$. The exponent, therefore, tells us how many factors of x there are following the 1. Consequently we could use this interpretation to say that x^0 means there are no factors of x to follow the 1; but, this fact does not mean that the answer is 0, however, because even though there are no factors of x there is still the 1 that was the coefficient. A further advantage to this kind of argument is that it can also be extended to generate a *meaning* for negative exponents. For example, if an exponent of 3 on a base of x tells us that we have 3 factors of x being multiplied by 1 (that is, $x^3 = 1.x.x.x$), then an exponent of -3 (the opposite to 3) ought to tell us we have 3 factors of x being divided (using the opposite operation to multiplication) into 1. So in this case x^{-3} would mean

$$(((1 \div x) \div x) \div x) = \left(\left(\frac{1}{x} \right) \div x \right) \div x = \left(\frac{1}{x} \times \frac{1}{x} \right) \div x = \frac{1}{x^2} \div x = \frac{1}{x^2} \times \frac{1}{x} = \frac{1}{x^3}. \quad \text{In fact this}$$

way of thinking does produce the correct value for an expression with a negative exponent. There are many other ways to *demonstrate* this result but this way is the only one that I am aware of that gives students a way of thinking about the *meaning* of negative exponents in the same way that they can think about the *meaning* of positive exponents. Now the overall understanding of exponents does not require a *special* status for zero or negative exponents because the meaning for these exponents has been directly connected to the meaning for whole number exponents.

During the interview with Julie, I asked a number of questions designed to encourage her to think about this lesson and the pupils' difficulties as a search for meaning. As noted above, she did realize that the base 2 was not a good choice but she was not able to establish how zero and negative exponents are consistent with the more familiar positive case. This somewhat disconnected knowledge of mathematical facts and principles is not unusual. The results of a large scale study of 127 pre-service teachers showed that prospective secondary school teachers (regardless of how intensive their academic preparation had been) were unable to make the kind of connection between positive and negative exponents as outlined above (Wood, 1993a). Many *were* able to suggest the patterning approach that Julie tried to use (with mixed success) but in all cases the primary goal was to get pupils to the stage where they could evaluate and rearrange expressions with exponents but not to establish a meaning for the symbols with which they were confronted. It seems clear that the knowledge which lies at the intersection of pedagogy and content is crucial for effective teaching. What is unclear is where teachers might develop and acquire this kind of knowledge; and, how they can be encouraged to do so.

In the next section I shall examine what differentiates the pedagogical knowledge of teacher educators (PCK) from that of teachers (pck) and the role that teacher education has to play in the development of pck in prospective mathematics teachers.

Pedagogical Content Knowledge of Teacher Educators

The development of the kind of pck discussed in the previous section makes a significant demand on the knowledge base of the teacher educator. In this case, the content for the teacher educator is the pck for the student teacher. The pedagogy consists of the teaching strategies that are used to develop this level of understanding and the intersection of the two becomes PCK for the teacher educator. This transformative component requires that the teacher educator understand how the student teachers think about the content and how they might be

brought to think about it in a different way. And, it also requires that the student teachers come to understand the pedagogical aspects of the knowledge that they are developing.

One way that these goals can be accomplished is by using teaching vignettes which allow for a wide ranging discussion of pedagogy and content and how they interact, while leaving the situation nested in a pedagogical setting. The discussion that ensues can be orchestrated by the teacher educator to bring out those aspects of content, pedagogy and pck that are important in this teaching situation. Furthermore, nesting such discussions in a teaching situation reduces the perception by the student teachers that their knowledge of elementary mathematics is being called into question—a perception that can cause both anxiety and resistance.

Vignettes such as the following one have been successfully used for this purpose:⁶

Teaching Vignette - Integral exponents

The following conversation takes place between a teacher and student in grade 9:

S: It doesn't make sense!

T: What's the matter?

S: Well you just taught us that $a^0 = 1$.

T: Is there something that you didn't understand in my explanation?

S: Well I thought I understood until I started to think about it.

T: Explain what's confusing you.

S: Well yesterday we learned that x^4 meant that you had 4 x 's, x cubed meant that you had three x 's and x squared meant that you had 2 x 's didn't we?

T: Yes.

S: So if you have x^0 doesn't that mean that you have zero x 's and so the answer should be zero?

T: Hm...

How would you respond as a teacher in this situation?

The discussion that would then follow the introduction of this vignette would seek to bring out the points discussed in previous sections with respect to content and pedagogy and how the two are linked together.

⁶See Wood (1993a) for other examples of teaching vignettes that have been successfully used to stimulate student teachers' thinking about pedagogical content knowledge.

In some cases, however, the PCK needed by teacher educators is very little different from the level of pck that they are trying to develop in their students because the way that prospective teachers think about the mathematical content in question is the same as the way that typical high school students think about the same content. Often, even very well prepared students still do not believe that $0.\overline{9} = 1$; and, their explanations of why the two expressions should or should not be the same resemble those of high school students (Wood, 1993a).

It seems that although most prospective teachers know their content, there appears to be little tendency to link what they have learned in university mathematics courses with what they teach in high school. It is only as they grapple with non trivial *pedagogical* issues that they begin to realize that their own knowledge and understandings may only be instrumental and not sufficiently deep for the purposes of teaching. It appears that the dichotomous nature of school mathematics and university mathematics does not encourage the students to reflect on and revisit their own understandings of high school mathematics after they have finished their high school experience. The overall result is that their understanding of high school mathematics is not much changed by advanced study at the university level.

The PCK of teacher educators, therefore, starts with an understanding of how their pre-service students typically think about mathematical ideas and the weaknesses inherent in those understandings. Teacher educators cannot assume that discussions of content are not part of their mandate; however, neither must they use these perceived deficiencies as a license to teach high school level content and call it teacher education! The focus must be on the *pedagogical understanding of content*, not simply a rehashing of an instrumental understanding of low level skills that students have already acquired in high school and university.

Pedagogical devices such as teaching vignettes and the development of good pedagogical cases with a mathematical basis can help provide prospective teachers with a framework for discussion of both pedagogy and mathematics at the same time. This approach can allow for a conceptual change in both these mathematical understandings and in students' views about how mathematics can be taught. These two dimensions must then be melded together to develop appropriate pck of the mathematics itself. This kind of teacher preparation makes substantial demands on teacher educators. However, it is crucial to address these issues at the intersection of content and pedagogy if the level of subject matter understanding that is necessary for innovative teaching is to be developed in prospective teachers.

References

- Anderson, C. W. (1989). The role of education in the academic disciplines in teacher education. In A. Woolfolk (Ed.), *Research perspectives on the graduate education of teachers* (pp. 88-107). Englewood Cliffs, NJ: Prentice Hall.
- Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *The Elementary School Journal*, *93*, 373-397.
- Fuller, R. (1997). Elementary teacher's pedagogical content knowledge of mathematics. *Mid Western Educational Researcher*, *10*(2), 9-16.
- Geddis, A. N. (1993). Transforming subject matter knowledge: The role of pedagogical content knowledge in learning to reflect on teaching. *International Journal of Science Education*, *15*, 673-683.
- Geddis, A. N., & Wood, E. F. (1997). Transforming subject matter and managing dilemmas. *Teaching and Teacher Education*, *13*, 611-626.
- Kinach, B. M. (2002). A cognitive strategy for developing pedagogical content knowledge in the secondary mathematics methods course: Toward a model of effective practice. *Teaching and Teacher Education*, *18*, 51-71.
- Lampert, M. (1986). Knowing, doing and teaching multiplication. *Cognition and Instruction*, *3*, 305-342.
- Lowrey, N. V. (2002). Construction of teacher knowledge in context: Preparing elementary teachers to teach mathematics and science. *School Science and Mathematics*, *102*, 68-83.
- Marks, R. (1990). Pedagogical content knowledge: From a mathematical case to a modified conception. *Journal of Teacher Education*, *41*, 3-11.
- McDiarmid, G. W., Ball, D. L., & Anderson, C. W. (1989). Why staying one chapter ahead doesn't really work: subject-specific pedagogy. In M. Reynolds (Ed.), *Knowledge base for beginning teachers*. New York: Pergamon.
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- Onslow, B., Beynon, C., & Geddis, A. (1992). Developing a teaching style: A dilemma for student teachers. *The Alberta Journal of Educational Research*, *38*, 301-315.
- Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*, 1-22.
- Wood, E. F. (1993a). *Making the connections: The mathematical understandings of prospective secondary teachers*. Unpublished doctoral dissertation, Michigan State University, Lansing, MI.

- Wood, E. F. (1993b). *Knowledge of nuggets or fragments of facts: How prospective mathematics teachers make connections*. Paper presented at the meeting of the Canadian Society for Studies in Education, Ottawa, ONT, June 10-13.
- Wood, E. F. (1993c). *Factors affecting the ways in which undergraduate mathematics education students explain school mathematics ideas*. Paper presented at the meeting of the Canadian Mathematics Educators Study Group, Toronto, ONT, May 28 - June 1.
- Wood, E. F. (1994). Connected Knowledge in Prospective Secondary Mathematics Teachers. In M. Quigley (Ed.), *Proceedings of the Seventeenth Annual Meeting of the Canadian Mathematics Educators Study Group*, Toronto, ONT, May 28 - June 1, 1993, (pp. 109-118).
- Wood, E., & Geddis, A. N. (1999). Self-conscious narrative and mathematics teacher education: Representing practice in professional course work. *Teaching and Teacher Education*, 15, (107-119).

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